Formal Languages and Linguistics

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Overview

Example I

$$
\begin{array}{lcl}
S & \rightarrow & A\,B \\
A & \rightarrow & aA \\
 & | & b \\
B & \rightarrow & bBc \\
 & | & \varepsilon
\end{array}
$$

- Rewriting system
- Auxiliary vocabulary (N for non-terminal)
- Start symbol (engendered language)
- Multiple derivations
- Syntactic tree

$S \to AB$

$A \to aA$

$A \to b$

$B \to bBc$

$\quad | \; \varepsilon$

$A \to a^n b$

$B \to b^m c^m$

$S \to AB$

$S \to AB \overset{*}{\to} b$

$S \overset{*}{\to} ab$

$\alpha = \{b, ab, aab, \ldots$

$\alpha = \{a^n b b^m c^m \mid n, m \in \mathbb{N}\}$

Example II

$$
\begin{array}{rcl}
E & \rightarrow & E + E \\
 & | & E \times E \\
 & | & (\, E\,) \\
 & | & 0 \mid 1 \mid 2 \ldots 8 \mid 9
\end{array}
$$

- Syntactic ambiguity
- Semantic interpretation

$E \to E + E$

$E \to E \times E$

$E \to (E)$

$E \to 0 | 1 | 2 | 3 \ldots 9$

$\Sigma = \{0, 1, 2 \ldots, +, \times, (,)\}$

$N = \{E\}$

~~$2 \times \times 3$~~ ~~$(((3)))$~~

$2 \times 3 + 1$

$E \to E \times E \to E \times E + E \to 2 \times E + E \to 2 \times 3 + E \to 2 \times 3 + 1$

$E \to E \times E \to E \times E + E \to E \times E + 1 \to 2 \times E + 1 \to 2 \times 3 + 1$

<u>left-derivation</u>

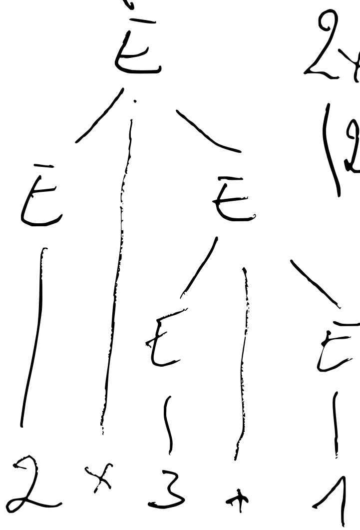
$E \to E \times E \to 2 \times E \to 2 \times E + E \to 2 \times 3 + E \to 2 \times 3 + 1$

$E \to E + E \to E \times E + E \to 2 \times E + E \to 2 \times 3 + E \to 2 \times 3 + 1$

$$E \to E \times E \to 2 \times E \to 2 \times E + E \to 2 \times 3 + E \to 2 \times 3 + 1$$

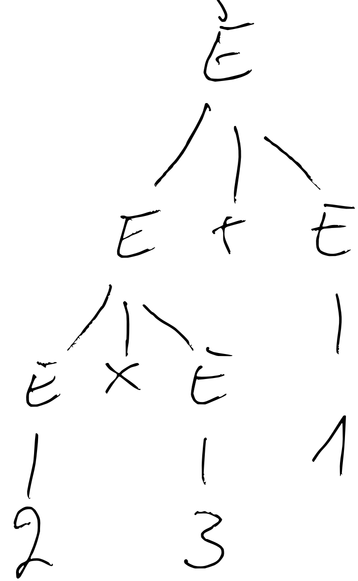
$$E \to E + E \to E \times E + E \to 2 \times E + E \to 2 \times 3 + E \to 2 \times 3 + 1$$

$$2 + 7 \times (3 + 1)$$



$$2 \times 3 + 1 = 7$$

$$(2 \times 3) + 1 = 7$$



$$E \to (E + E)$$

$$E \to (E \times E)$$

$$E \to 0 | 1 \dots 9$$

$$3 + 7 \times 2$$

E T F

$$E \to E + T \mid T$$

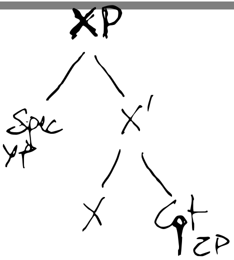
$$T \to T \times F \mid F$$

$$F \to (E)$$

$$\mid 0 | 1 \dots 9$$

X : N

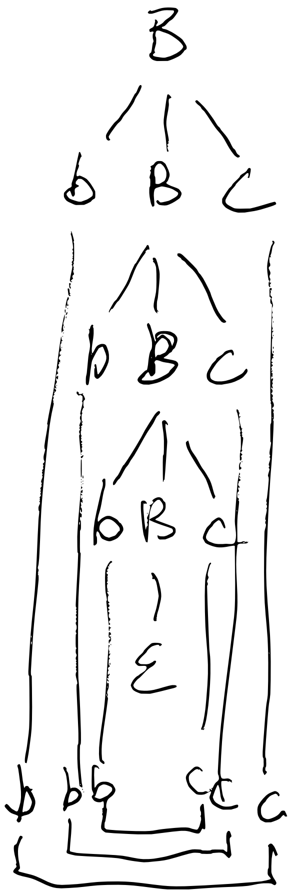
Example III



NP	$\rightarrow$	*Det N′*
N′	$\rightarrow$	*AdjP N′*
N′	$\rightarrow$	*N*
N′	$\rightarrow$	*N Cpt*
AdjP	$\rightarrow$	*Adj AdjP*
AdjP	$\rightarrow$	*Adj*
Cpt	$\rightarrow$	*P NP*
Det	$\rightarrow$	the | my
N	$\rightarrow$	cat | friend
Adj	$\rightarrow$	large | fierce
Prep	$\rightarrow$	of | to

- X-bar theory
- Recursive rules
- Center-embedding

the fierce friend of my cat


B
b B C
b B C
b B C
ε
b b b c c c

Overview

Formal grammar

$S \to aSb \mid \varepsilon$

$S \to S$

$BB \to AB$

$S \to aBc$

$aSa \to aBB$

~~$adc \to B$~~

Def. 12 ((Formal) Grammar)

A **formal grammar** is defined by $\langle \Sigma, N, S, P \rangle$ where

- Σ is an alphabet
- N is a disjoint alphabet (non-terminal vocabulary)
- $S \in N$ is a distinguished element of N, called the *axiom*
- P is a set of « *production rules* », namely a subset of the cartesian product $(\Sigma \cup N)^* N (\Sigma \cup N)^* \times (\Sigma \cup N)^*$.

(BB, AB)

Immediate Derivation

Def. 13 (Immediate derivation)

Let $\mathcal{G} = \langle \Sigma, N, S, P \rangle$ a grammar,
$r \in P$ a production rule, such that $r : A \longrightarrow u$ with $u \in (\Sigma \cup N)^*$;
$f, g \in (\Sigma \cup N)^*$ two "(proto-)words",

- f derives into g (immediate derivation) with the rule r (noted $f \xrightarrow{r} g$) iff
  $\exists v, w$ s.t. $f = vAw$ and $g = vuw$
- f derives into g (immediate derivation) in the grammar $\mathcal{G}$ (noted $f \xrightarrow{\mathcal{G}} g$) iff
  $\exists r \in P$ s.t. $f \xrightarrow{r} g$.

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Derivation

Def. 14 (Derivation)

$f \xrightarrow{\mathcal{G}*} g$ if $f = g$ or

$\exists f_0, f_1, f_2, ..., f_n$ s.t.

$f_0 = f$

$f_n = g$

$\forall i \in [1, n] : f_{i-1} \xrightarrow{\mathcal{G}} f_i$

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Engendered language

Def. 15 (Language engendered by a word)

Let $f \in (\Sigma \cup N)^*$.

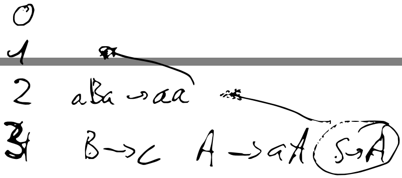
$L_{\mathcal{G}}(f) = \{g \in \Sigma^* / f \xrightarrow{\mathcal{G}*} g\}$

Def. 16 (Language engendered by a grammar)

The *language engendered by a grammar* $\mathcal{G}$ is the set of words of Σ^* derived from the axiom.

$L_{\mathcal{G}} = L_{\mathcal{G}}(S)$

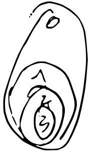
Overview



Principle

Define language families on the basis of properties of the grammars that generate them :

1. Four classes are defined, they are included one in another
2. A language is of type k if it **can** be recognized by a type k grammar (and thus, by definition, by a type $k-1$ grammar) ; and **cannot** be recognized by a grammar of type $k+1$.



Chomsky's hierarchy

Schützenberger

0 $aSb \to c$
1 $aSb \to acSb$
2 $S \to aSb$
3 $A \to aB$, $A \to b$, $A \to \varepsilon$

type 0 No restriction on $P \subset (X \cup V)^* V (X \cup V)^* \times (X \cup V)^*$.

type 1 (*context-sensitive* grammars) All rules of P are of the shape $(u_1 S u_2, u_1 m u_2)$, where u_1 and $u_2 \in (X \cup V)^*$, $S \in V$ and $m \in (X \cup V)^+$.

type 2 (*context-free* grammar) All rules of P are of the shape (S, m), where $S \in V$ and $m \in (X \cup V)^*$.

type 3 (*regular* grammars) All rules of P are of the shape (S, m), where $S \in V$ and $m \in X.V \cup X \cup \{\varepsilon\}$.

$u_1 S u_2 \to u_1 m u_2$ $A \to aA$

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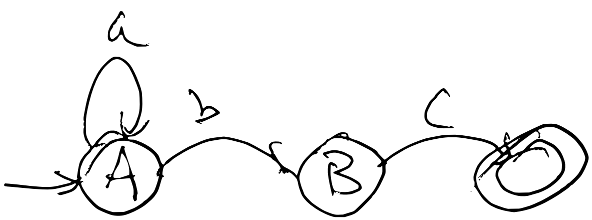
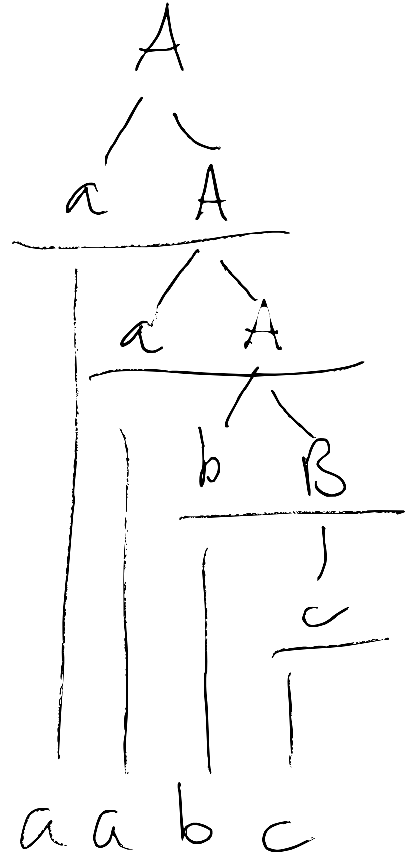
axiom: ~~A~~ S

$A \to aA$
$\quad | \; bB$

$S \to A$
$\quad | \; B$

$B \to c$





Examples

type 3:

$$\begin{array}{lcl} S & \rightarrow & aS \mid aB \mid bB \mid cA \\ B & \rightarrow & bB \mid b \\ A & \rightarrow & cS \mid bB \end{array}$$

Examples

type 3:

$$\begin{array}{lcl} S & \rightarrow & aS \mid aB \mid bB \mid cA \\ B & \rightarrow & bB \mid b \\ A & \rightarrow & cS \mid bB \end{array}$$

type 2:

$E \rightarrow E + T \mid T, T \rightarrow T \times F \mid F, F \rightarrow (E) \mid a$

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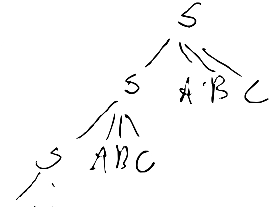
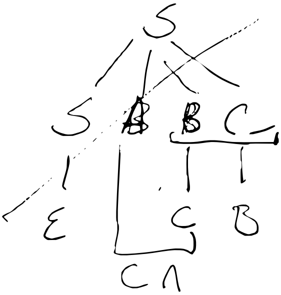
Example 1 type 0





Type 0:

$S \rightarrow SABC$	$AC \rightarrow CA$	$A \rightarrow a$
$S \rightarrow \varepsilon$	$CA \rightarrow AC$	$B \rightarrow b$
$AB \rightarrow BA$	$BC \rightarrow CB$	$C \rightarrow c$
$BA \rightarrow AB$	$CB \rightarrow BC$	

generated language :

$S \rightarrow SABC \rightarrow ABC \rightarrow BAC \rightarrow BCA \xrightarrow{*} bca$

Example 1 type 0

Type 0:

$$\begin{array}{lll} S \rightarrow SABC & AC \rightarrow CA & A \rightarrow a \\ S \rightarrow \varepsilon & CA \rightarrow AC & B \rightarrow b \\ AB \rightarrow BA & BC \rightarrow CB & C \rightarrow c \\ BA \rightarrow AB & CB \rightarrow BC & \end{array}$$

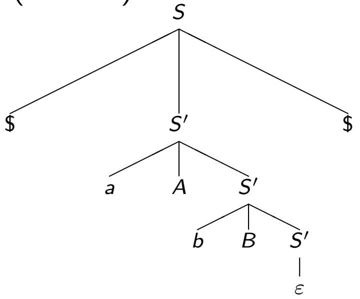
generated language : words with an equal number of a, b, and c.

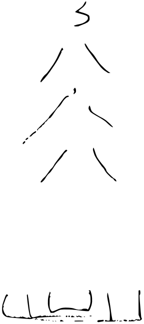
Example 2: type 0

Type 0:

$$
\begin{array}{llll}
S \to \$S'\$ & Aa \to aA & \$a \to a\$ \\
S' \to aAS' & Ab \to bA & \$b \to b\$ \\
S' \to bBS' & Ba \to aB & A\$ \to \$a \\
S' \to \varepsilon & Bb \to bB & B\$ \to \$b \\
 & & \$\$ \to \#
\end{array}
$$

Example 2: type 0 (cont'd)

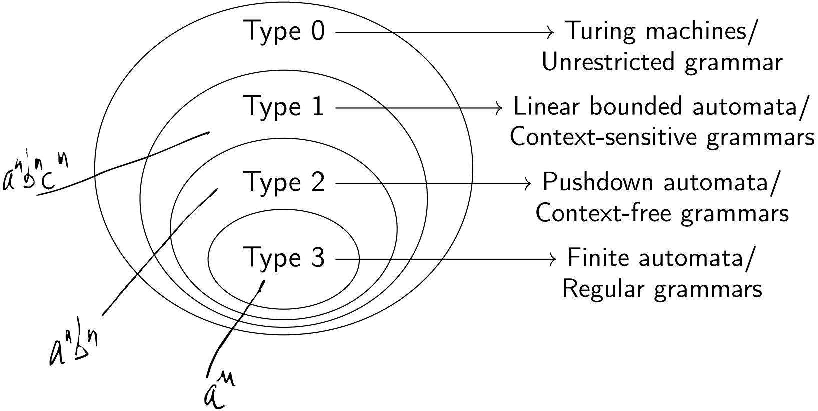




$	a	A	b	B	$
a	$	A	b	B	$
a	$	A	b	$	b
a	$	b	A	$	b
a	b	$	A	$	b
a	b	$	$	a	b
a	b	#		a	b

$S \rightarrow aSb \mid \varepsilon$

The Chomsky-Schützenberger hierarchy



Remarks

- Type 0 (Turing-recognizable) = recursively enumerable languages
  Type 1 (Turing-decidable) = recursive languages
- There are others ways to classify languages,
  - either on other properties of the grammars;
  - or on other properties of the languages
- Nested structures are preferred, but it's not necessary

The parsing problem: finding derivations

- Given a grammar G on some alphabet Σ...
- The **parsing problem** for G:

  Given some $w \in \Sigma$,
  what are the derivations (if any) of w in G?
- (Solving the parsing problem for G entails solving the recognition problem for $\mathcal{L}(G)$.)

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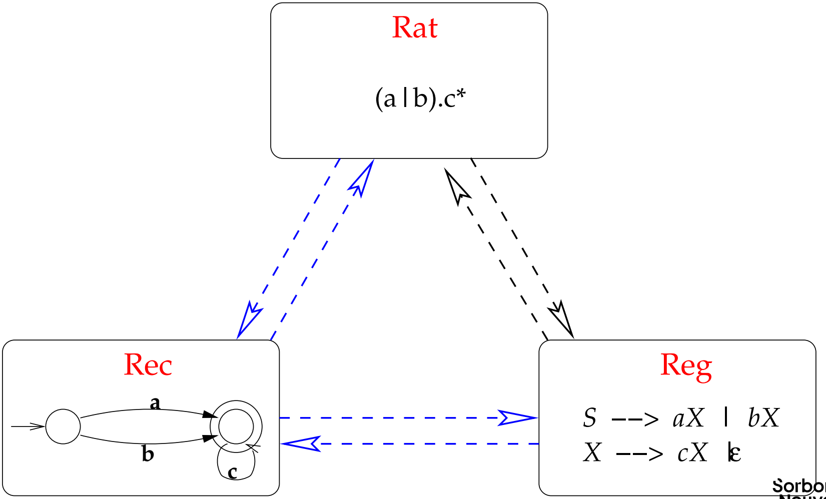
Syntactic complexity vs semantic expressivity

- Context-free grammars are commonly used to describe the syntax of many logical languages (PL, FOL), some programming languages, and parts of NL ($\rightarrow$ Day 2).
- Untyped λ-calculus: CF syntax, Turing-complete semantics. "How is this possible?"
- $\rightarrow$ The syntactic complexity and the semantic expressivity of interpreted languages are two distinct notions.
- Jot (https://en.wikipedia.org/wiki/Iota_and_Jot) is $\{0, 1\}$, a regular language, compositionally interpreted as a Turing-complete language.

The recognition/parsing problems are very general

- Consider any binary ("yes/no") problem P and see it as the set of inputs for which the answer is positive.
- Let *str* be a linearisation function for the possible inputs of P, and $L = \{str(in) \mid in \in P\}$.
- Solving P is equivalent to the recognition problem for L.
- More generally, any computable function f can be encoded as a grammar s.t. after parsing the input w, the output $f(w)$ can be read off the derivation.
- $\rightarrow$ One can compute "syntactically": a grammar is a program. (The parser is the machine that runs it.)
- The formalism of unrestricted grammars is a Turing-complete programming language. (syntactically regular?)

Back to regular languages



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Let's play with grammars

For each of the following grammars, give the generated language, and the type they have in Chomsky's hierarchy.

$$\begin{array}{lcl} S & \rightarrow & S_1 S_2 \\ S_1 & \rightarrow & aS_1 b \mid ab \\ S_2 & \rightarrow & cS_2 \mid c \end{array}$$

$$\begin{array}{lcl} S & \rightarrow & aSBC \\ S & \rightarrow & aBC \\ CB & \rightarrow & BC \\ aB & \rightarrow & ab \\ bB & \rightarrow & bb \\ bC & \rightarrow & bc \\ cC & \rightarrow & cc \end{array}$$

Let's play with grammars (cont'd)

Give a contex-free grammar that generates each of the following languages (alphabet $\Sigma = \{a, b, c\}$).

- $L_0 = \{w \in X^* \;/\; w = a^n \;;\; n \geq 0\}$
- $L'_0 = \{w \in X^* \;/\; w = a^n b^n ca \;;\; n \geq 0\}$
- $L_1 = \{w \in X^* \;/\; w = a^n b^n c^p ; n > 0 \text{ et } p > 0\}$
- $L_2 = \{w \in X^* \;/\; w = a^n b^n a^m b^m ; n, m \geq 1\}$
- $L'_3 = \{w \in X^* \;/\; |w|_a = |w]_b\}$
- $L_3 = \{w \in X^* \;/\; |w|_a = 2|w]_b\}$
- $L_4 = \{w \in X^* \;/\; \exists x \in X^* \text{ tq } w = x\overline{x}\}$
- $L_5 = \{w \in X^* \;/\; w = \overline{w}\}$

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