Ex. 1

Consider the following sentences.

(1) a. Alice eats cakes.
b. The caterpillar gives Alice cakes.
c. The cat with a grin disappears.
d. Alice paints white roses red.

Define a context-free grammar that could generate these sentences.

Ex. 2

1. Let G be the grammar $S \to aSbb \mid \varepsilon$. Describe informally the language generated by G.
2. Let G' be the grammar $S' \to SSS$, $S \to aSbb \mid \varepsilon$, with S' as the start symbol (axiom). Describe informally the language generated by G'.
3. Let G_1 and G_2 be context-free grammars; $L(G_1)$ and $L(G_2)$ the languages they generate. Show that there is a context-free grammar generating each of the following sets:
   (a) $L(G_1) \cup L(G_2)$
   (b) $L(G_1)L(G_2)$
   (c) $L(G_1)^*$