
 Exercice 1

Syntax of λ -terms Are the following λ -terms well formed?

- (5) a. $(x)x.x$
 b. $\lambda x.\lambda y.\lambda z.u$
 c. $\lambda y.(\lambda x.(y))x$
 d. $\lambda x.(xx)$
 e. $(x)\lambda y.x$

.....Answer.....

- (6) a. $(x)x.x$
 not well formed : a dot may appear only in connexion with a lambda symbol
 b. $\lambda x.\lambda y.\lambda z.u$
 well-formed
 c. $\lambda y.(\lambda x.(y))x$
 well-formed
 d. $\lambda x.(xx)$
 not well formed : for every functional application we need to have a functional term between parenthesis and an operand on its right. The notation (xy) is however a possible alternative way to mark functional application.
 e. $(x)\lambda y.x$
 well-formed

 Exercice 2

β -reduction : Reduce as much as possible the following λ -terms

- (7) a. $(\lambda x.(x)x)\lambda x.x$
 b. $((\lambda x.\lambda y.(y)x)f)\lambda x.x$
 c. $(\lambda n.\lambda f.\lambda x.(f)((n)f)x)\lambda(f)x.(f)x$

.....Answer.....

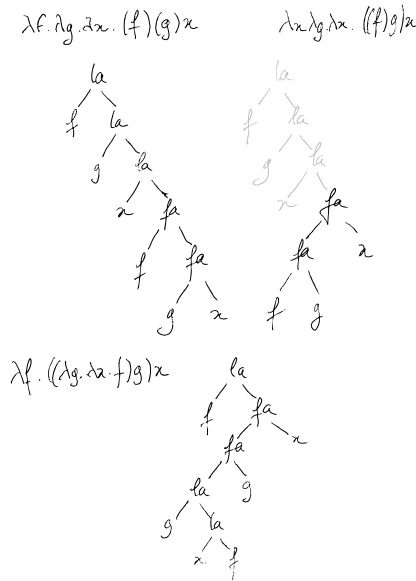
- (8) a. $(\lambda x.(x)x)\lambda x.x$
yields $\lambda x.x$ after 2 β -reductions
 b. $((\lambda x.\lambda y.(y)x)f)\lambda x.x$
yields f after 3 β -reductions
 c. $(\lambda n.\lambda f.\lambda x.(f)((n)f)x)\lambda(f)x.(f)x$
this term is not well-formed

Exercise 3

Syntax of λ -terms Represent the following terms as (syntactic) trees.

- (9) a. $\lambda f.\lambda g.\lambda x.(f)(g)x$
- b. $\lambda f.\lambda g.\lambda x.((f)g)x$
- c. $\lambda f.((\lambda g.\lambda x.f)g)x$

..... Answer



Exercise 4

Redex & β -reduction Identify all redexes in the following term, and reduce it as much as possible.

- (10) $((\lambda S.\lambda V.(S)(V)\lambda Q.(Q)m)\lambda P.(P)j)\lambda O.\lambda y.(O)\lambda z.((kiss)y)z$

..... Answer

Let's write K instead of $kiss$.

Redexes are underlined like this : $(\lambda x.M)\underline{N}$

$$\begin{aligned}
 & ((\lambda S.\lambda V.(S)(V)\lambda Q.(Q)m)\lambda P.(P)j)\lambda O.\lambda y.(O)\lambda z.((K)y)z \\
 & \underline{(\lambda V.(\lambda P.(P)j)(V)\lambda Q.(Q)m)\lambda O.\lambda y.(O)\lambda z.((K)y)z} \\
 & (\lambda P.(P)j)\underline{(\lambda O.\lambda y.(O)\lambda z.((K)y)z)}\lambda Q.(Q)m \\
 & (\lambda P.(P)j)\lambda y.(\lambda Q.(Q)m)\lambda z.((K)y)z \\
 & (\lambda y.(\lambda Q.(Q)m)\lambda z.((K)y)z)\underline{j} \\
 & (\lambda Q.(Q)m)\lambda z.((K)j)z \\
 & (\lambda z.((K)j)z)\underline{m} \\
 & ((K)j)m \\
 & Kjm
 \end{aligned}$$

Exercice 5

Notation conventions

Since dot + parenthesis notation can become rather heavy, the following conventions are often adopted :

$$\begin{aligned}\lambda x_1.\lambda x_2 \dots \lambda x_n.t &= \lambda x_1 x_2 \dots x_n.t \\ t(t_1)(t_2) \dots (t_m) &= t t_1 t_2 \dots t_m\end{aligned}$$

Example : $\lambda xy.xy$ is read $\lambda x.\lambda y.x(y)$.

Note : Under this convention, the notation abc is not ambiguous : it corresponds to the term $((a)b)c$, or $fa(fa(a, b), c)$. To express the (different) term $fa(a, fa(b, c))$ at least one pair of parenthesis has to be inserted $(a)bc$.

Propose fully parenthesized versions of the following terms. If they can be reduced, reduce them.

- (11) a. $\lambda xz.xyz$
 b. $(\lambda x.\lambda y.fxy)xy$
 c. $(\lambda x.\lambda y.xyy)\lambda y.\lambda a.y$

.....Answer.....

- (12) a. $\lambda xz.xyz$
 $\lambda x.\lambda y.((x)y)z$ *not reducible*
 b. $(\lambda x.\lambda y.fxy)xy$
 $(\lambda x.\lambda y.((f)x)y)(x)y$ *which yields $\lambda y.((f)(x)y)y$*
 c. $(\lambda x.\lambda y.xyy)\lambda y.\lambda a.y$
 $(\lambda x.\lambda y.((x)y)y)\lambda y.\lambda a.y$ *which yields $\lambda y.y$*